STA237 Tutorial 7

Kevin Dang

University of Toronto

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Information

- Email: kevinquan.dang@mail.utoronto.ca
- Website: dang-kevin.github.io/sta237
- Office hours: Tuesdays 1-2pm
- For all Tutorial/Assignment/Test grading inquiries: contact Professor Selvaratnam at sta237@utoronto.ca

Agenda

1. Review of key concepts
2. Tutorial Problems
3. Q&A

Uniform Distribution

Definition

If $a < b$, a random variable Y is said to have a continuous uniform probability distribution on the interval (a, b) if and only if the density function of Y is

$$f(y) = \begin{cases} \frac{1}{b-a}, & a \leq y \leq b \\ 0, & \text{elsewhere} \end{cases}$$

The Method of Distribution Functions

If Y has probability density function $f(y)$ and if U is some function of Y, then we can find $F_U(u) = P(U \leq u)$ directly by integrating $f(y)$ over the region for which $U \leq u$. The probability density function for U is found by differentiating $F_U(u)$.

Summary of the Distribution Function Method

Let U be a function of the random variables $Y_1, Y_2, \cdots, Y_n$.

1. Find the region $U = u$ in the $(y_1, y_2, \cdots, y_n)$ space.
2. Find the region $U \leq u$.
3. Find $F_U(u) = P(U \leq u)$ by integrating $f(y)$ over the region $U \leq u$.
4. Find the density function $f_U(u)$ by differentiating $F_U(u)$. Thus, $f_U(u) = dF_U(u)/du$.

Instructions

- You will receive an email at the end of the tutorial session to upload your work. Also, you will know that which question should be uploaded at that time.
- You will have **4 hours window** to upload your work.
- If you upload the work of others on your Crowdmark link, you will get maximum 10% penalty in your course marks.
- **You should only upload one question that will be instructed on Crowdmark**

Question 1

Let Y be a random variable with probability density function given by

$$f(y) = \begin{cases} 2(1-y), & 0 \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the density function of $U_1 = 2Y - 1$.

(b) Find the density function of $U_2 = 1 - 2Y$.

(c) Find the density function of $U_3 = Y^2$.

Question 2 (Not Covered)

Consider a random variable Y_1, the proportion of impurities in a chemical sample, and Y_2, the proportion of type 1 impurities among all impurities in the sample. The joint density function was given by

$$f(y_1, y_2) = \begin{cases} 2(1-y), & 0 \leq y_1 \leq y_2 \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

We are interested in $U = Y_1 Y_2$, which is the proportion of type 1 impurities in the sample. Find the probability density function for U and use it to find $E(U)$.