STA237 Tutorial 6

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Information

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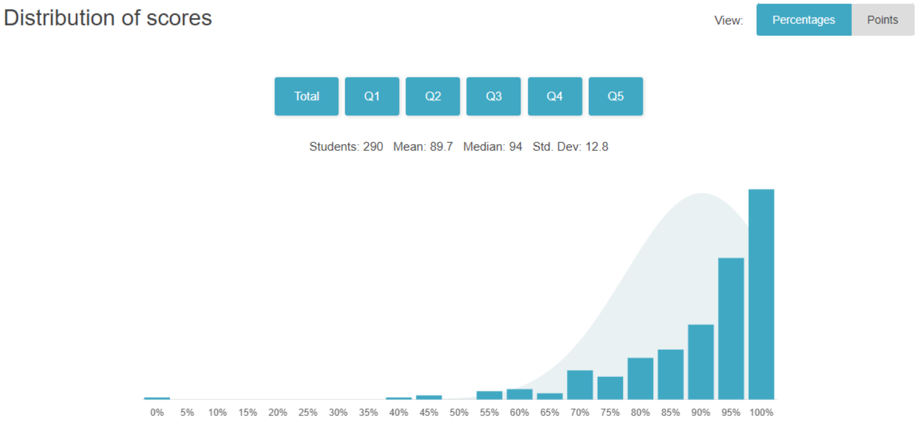
- Office hours: Tuesdays 1-2pm
- For all Tutorial/Assignment/Test grading inquiries: contact Professor Selvaratnam at sta237@utoronto.ca

Midterm Distribution: LEC0101



Agenda

1. Review of key concepts
2. Tutorial Problems
3. Q&A

Binomial Distribution

- $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$
- $\binom{n}{x} = \frac{n!}{x!(n-x)!}$
- $X \sim Bin(n, p)$
  - $E(X) = \mu = np$
  - $Var(X) = \sigma^2 = np(1-p)$
- When n is sufficiently large, we can use the normal approximation to the binomial
  - $X \sim N(np, np(1-p))$
  - $\frac{X-np}{\sqrt{np(1-p)}} \sim N(0,1)$

Uniform Distribution

Definition

If $a < b$, a random variable Y is said to have a continuous uniform probability distribution on the interval (a, b) if and only if the density function of Y is

$$f(y) = \begin{cases} \frac{1}{b-a}, & a \leq y \leq b \\ 0, & \text{elsewhere} \end{cases}$$

Discrete

Let Y_1 and Y_2 be discrete random variables. The joint distribution function $F(y_1, y_2)$ is

$$F(y_1, y_2) = P(Y_1 \leq y_1, Y_2 \leq y_2), -\infty < y_1, y_2 < \infty$$

Continuous

Let Y_1 and Y_2 be continuous random variables with joint distribution function $F(y_1, y_2)$. If there exists a nonnegative function $f(y_1, y_2)$, such that

$$F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2) dt_2 dt_1,$$

for all $-\infty < y_1, y_2 < \infty$, then Y_1 and Y_2 are said to be joint continuous random variables. The function $f(y_1, y_2)$ is called the the joint probability density function.

Instructions

- You will receive an email at the end of the tutorial session to upload your work. Also, you will know that which question should be uploaded at that time.
- You will have **4 hours window** to upload your work.
- If you upload the work of others on your Crowdmark link, you will get maximum 10% penalty in your course marks.
- **You should only upload one question that will be instructed on Crowdmark**

Question 1

Let X be the number of heads in 10,000 coin tosses. We expect $X \approx 5000$. What range of values of X are typically observed with high probability, say 0.99? More precisely, what number t satisfies

$$P(5000 - t \leq X \leq 5000 + t) = 0.99?$$

Question 2

The joint density of Y_1, the proportion of the capacity of the tank that is stocked at the beginning of the week, and Y_2, the proportion of the capacity sold during the week, is given by

$$f(y_1, y_2) = \begin{cases} 3y_1, & 0 \le y_2 \le y_1 \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find $F(1/2, 1/3) = P(Y_1 \le 1/2; Y_2 \le 1/3)$.

(b) Find $P(Y_2 \le Y_1/2)$, the probability that the amount sold is less than half the amount purchased.

Question 3

A soft-drink machine has a random amount Y_2 in supply at the beginning of a given day and dispenses a random amount Y_1 during the day (with measurements in gallons). It is not resupplied during the day, and hence $Y_1 \leq Y_2$. It has been observed that Y_1 and Y_2 have a joint density given by

$$f(y_1, y_2) = \begin{cases} 1/2, & 0 \leq y_2 \leq y_1 \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

That is, the points $(y1, y2)$ are uniformly distributed over the triangle with the given boundaries.

(a) Find the conditional density of Y_1 given $Y_2 = y_2$.

(b) Evaluate the probability that less than $1/2$ gallon will be sold, given that machine contains 1.5 gallons at the start of the day.