STA237 Tutorial 4

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Information

- Email: kevinquan.dang@mail.utoronto.ca
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- Office hours: Tuesdays 1-2pm
- For all Tutorial/Assignment/Test grading inquiries: contact Professor Selvaratnam at sta237@utoronto.ca
- Midterm
  - Tuesday October 19, 3:00pm – 5:00pm
  - Thursday October 21, 6:00pm – 8:00pm

Agenda

1. Review of key concepts
2. Tutorial Problems
3. Q&A

Normal Distribution

- Density function of X: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$
- $E(X) = \mu$
- $Var(X) = \sigma^2$
- $X \sim N(\mu, \sigma^2)$
- $X = \sigma Z + \mu \Rightarrow Z = \frac{X-\mu}{\sigma}$

Properties

- The mean, mode and median are all equal.
- The curve is symmetric at the center (i.e. around the mean, μ).
- Exactly half of the values are to the left of center and exactly half the values are to the right.
- The total area under the curve is 1.

Normal Random Variables

Sum of independent normal random variables is normal

Let $X_1, X_2, \cdots, X_n$ be a sequence of independent normal random variables with

$$X_k \sim N(\mu_k, \sigma_k^2), k = 1, 2, \cdots, n$$

Then

$$X_1 + X_2 + \cdots + X_n \sim N(\mu_1 + \mu_2 + \cdots + \mu_n, \sigma_1^2 + \cdots + \sigma_n^2)$$

Instructions

- You will receive an email at the end of the tutorial session to upload your work. Also, you will know that which question should be uploaded at that time.
- You will have **4 hours window** to upload your work.
- If you upload the work of others on your Crowdmark link, you will get maximum 10% penalty in your course marks.
- **You should only upload one question that will be instructed on Crowdmark**

Question 1

The mass of a cereal box is normally distributed with mean 385g and standard deviation 5g. What is the probability that 10 boxes will contain less than 3800g of cereal?

Question 2 [Material not covered, moved to tutorial 5]

Let X be the number of heads in 10,000 coin tosses. We expect $X \approx 5000$. What range of values of X are typically observed with high probability, say 0.99? More precisely, what number t satisfies

$$P(5000 - t \leq X \leq 5000 + t) = 0.99?$$

Question 3

Suppose an automobile manufacturer introduces a new model that has an advertised mean in-city mileage of 27 miles per gallon. Although such advertisements seldom report any measure of variability, suppose you write the manufacturer for the details of the tests and you find that the standard deviation is 3 miles per gallon. This information leads you to formulate a probability model for the random variable x, the in-city mileage for this car model. You believe that the probability distribution of x can be approximated by a normal distribution with a mean of 27 and a standard deviation of 3.

a. If you were to buy this model of automobile, what is the probability that you would purchase one that averages less than 20 miles per gallon for in-city driving? In other words, find $P(x < 20)$.
b. Find $P(15 < x < 25)$.
c. Find $P(x > 50)$.