STA237 Tutorial 3

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Information

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Agenda

1. Review of key concepts
2. Tutorial Problems
3. Q&A

Binomial Distribution

- $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$
- $\binom{n}{x} = \frac{n!}{x!(n-x)!}$
- $E(X) = np$
- $Var(X) = np(1-p)$

Properties

- The experiments consists of a fixed number, n, of identical trials.
- Each trial results in one of two outcomes: success, S, or failure, F.
- The probability of success on a single trial is equal to some value p and remains the same from trial to trial. The probability of a failure is equal to $q = 1 - p$
- The trials are independent

Poisson Distribution

- $p(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \ \lambda > 0$
- $E(X) = Var(X) = \lambda$, where λ is the average number of events.
- x is the number of times an event occurs in an interval.
- Can be rewritten as $P(\text{x events in interval t}) = \frac{(rt)^x e^{-rt}}{x!}$, where r is number of events per unit of time. $(\lambda = rt)$

Properties

- Events are independent of each other. The occurrence of one event does not affect the probability another event will occur.
- The average rate (events per time period) is constant.
- Two events cannot occur at the same time.

Recursion

- The process in which a function calls itself directly or indirectly is called recursion.

Example: summing numbers from 1 to n

1. Standard approach
   - $f(n) = 1 + 2 + \cdots + n$
2. Recursion
   - $f(n) = 1 \qquad n = 1.$
   - $f(n) = n + f(n-1) \qquad n > 1$

Instructions

- You will receive an email at the end of the tutorial session to upload your work. Also, you will know that which question should be uploaded at that time.
- You will have **4 hours window** to upload your work.
- If you upload the work of others on your Crowdmark link, you will get maximum 10% penalty in your course marks.
- **You should only upload one question that will be instructed on Crowdmark**

Recursive Programming

- Recursive programming is a powerful programming techniques, made possible by functions. A recursive program is simply one that calls itself. This is useful because many algorithms are recursive in nature.

Question 1

We can write $n!$ as $n * ((n-1)!)$. Write a program to compute $n!$ by implementing a recursive programming technique.

Question 2

A particular concentration of a chemical found in polluted water has been found to be lethal to 20% of the fish that are exposed to the concentration for 24 hours. Twenty fish are placed in a tank containing this concentration of chemical in water.

a. Find the probability that exactly 14 survive.
b. Find the probability that at least 10 survive.
c. Find the probability that at most 16 survive.
d. Find the mean and variance of the number that survive.

Question 3

Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that

a. no more than three customers arrive?
b. at least two customers arrive?
c. exactly five customers arrive?

Question 4

a. Write R-codes to create a matrix $A = \begin{pmatrix} 3 & 4 & 6 & 1 \\ 4 & 9 & 7 & 6 \\ 2 & 3 & 2 & 9 \end{pmatrix}$

b. Find the inverse of AA^T