STA237 Tutorial 2

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Information

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- Website: dang-kevin.github.io/sta237
- Tutorial: Thursdays 3:10-5pm
- Office hours: Tuesdays 1-2pm
- Locations: Zoom (links posted on Quercus)
- For Tutorial 1 grading inquiries: contact Harold Lee at haroldhyun.lee@mail.utoronto.ca

Agenda

1. Review of key concepts
2. Tutorial Problems
3. Q&A

Discrete Random Variables

- Let X be a discrete random variable with the probability function $p(x)$. Then the expected value of X, $E(X)$, is defined to be

$$E(X) = \sum_x xP(x)$$

- If X is a random variable with mean $E(X) = \mu$, the variance of a random variable X is defined to be the expected value of $(X - \mu)^2$, i.e.

$$Var(X) = E[(X - \mu)^2]$$

Properties of expected value and variance of random variables

If X and Y are random variables and a and b are constants, we have

- $E(aX + bY) = aE(X) + bE(Y)$
- $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$ if X and Y are independent

Discrete Distributions

- Binomial distribution Bin(n,p)
  - $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$
  - $\binom{n}{x} = \frac{n!}{x!(n-x)!}$
  - $E(X) = np$
  - $Var(X) = np(1-p)$

Properties of Binomial distribution

- The experiments consists of a fixed number, n, of identical trials.
- Each trial results in one of two outcomes: success, S, or failure, F.
- The probability of success on a single trial is equal to some value p and remains the same from trial to trial. The probability of a failure is equal to $q = 1 - p$
- The trials are independent

Discrete Distributions

- Bernoulli distribution Ber(p)
  - $P(X = x) = \begin{cases} p & x = 1 \\ 1 - p & x = 0 \end{cases}$
  - $E(X) = p$
  - $Var(X) = p(1 - p)$
- Bernoulli is a special case of Binomial, when n = 1:
  - $P(X = x) = \binom{1}{x} p^x (1 - p)^{1-x} = p^x (1 - p)^{1-x}$
  - $P(X = 1) = p^1 (1 - p)^{1-1} = p$
  - $P(X = 0) = p^0 (1 - p)^{1-0} = 1 - p$

Instructions

- You will receive an email at the end of the tutorial session to upload your work. Also, you will know that which question should be uploaded at that time.
- You will have **4 hours window** to upload your work.
- If you upload the work of others on your Crowdmark link, you will get maximum 10% penalty in your course marks.
- **You should only upload one question that will be instructed on Crowdmark**

Question 1

Given independent random variables X and Y, with means and standard deviations as shown,

	Mean	SD
X	10	2
Y	20	5

find the mean and standard deviation of each of the random variables below. Also, X_1 and X_2 are independent variables with the same distribution as X.

i. $3X$
ii. $Y + 6$
iii. $X + Y$
iv. $X - Y$
v. $X_1 + X_2$

Question 2

Write a program in R to display the first n powers of the number x.

Question 3

i. Write a function in R to add two numbers x and y.

ii. Write a function in R to compute $n!$, where n is a positive integer.

Question 4

An insurance company issues a one-year \$1000 policy insuring against an occurrence A that historically happens to 2 out of every 100 owners of the policy. Administrative fees are \$15 per policy and are not part of the company's "profit". How much should the company charge for the policy if it requires that the expected profit per policy be \$50?