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Q1 (a) $U_1 = 2Y - 1$

$P(U_1 \leq u) = P(2Y - 1 \leq u) = P\left(Y \leq \frac{u+1}{2}\right)$

* From the density of Y, we know the support of this

$\frac{u+1}{2} < 0 \iff u < -1 \quad , P(U_1 < u) = 0$

$0 \leq \frac{u+1}{2} \leq 1 \iff -1 < u < 1 \quad , P(U_1 < u) = ?$

$\frac{u+1}{2} > 1 \iff u > 1 \quad , P(U_1 < u) = 1$

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→ on $-1 < u < 1$:

$$P\left(Y \le \frac{u+1}{2}\right) = \int_0^{\frac{u+1}{2}} 2(1-y)\,dy = 2y - y^2 \Big]_0^{\frac{u+1}{2}}$$

$$= 2\left(\frac{u+1}{2}\right) - \left(\frac{u+1}{2}\right)^2 = (u+1) - \frac{(u+1)^2}{4}$$

$$\therefore F_{u_1}(u) = \begin{cases} 0 & , u < -1 \\ (u+1) - \frac{(u+1)^2}{4} & , -1 \le u \le 1 \\ 1 & , u > 1 \end{cases} \qquad f_{u_1}(u) = \begin{cases} 1 - \left(\frac{u+1}{2}\right) & , -1 \le u \le 1 \\ 0 & , \text{else} \end{cases}$$

$$f_{u_1}(u) = \frac{d}{du} F_{u_1}(u)$$

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Q1(b) Follow same steps as Q1(a):

$$P(U_2 \le u) = P(1-2Y \le u) = P\left(Y \ge \frac{1-u}{2}\right)$$

$$P(Y \ge y) = \begin{cases} 0, & y > 1 \\ ?, & 0 \le y \le 1 \\ 1, & y < 0 \end{cases}$$

$1-u > 2$

$-1 > u$

$$\Rightarrow P(U_2 \le u) = \begin{cases} 0, & \frac{1-u}{2} > 1 \iff u < -1 \\ ?, & 0 \le \frac{1-u}{2} \le 1 \iff -1 \le u \le 1 \\ 1, & \frac{1-u}{2} < 0 \iff u > 1 \end{cases}$$

On $-1 \le u \le 1$

$$P(U_2 \le u) = P\left(Y \ge \frac{1-u}{2}\right) = \int_{\frac{1-u}{2}}^{1} 2(1-y)\,dy = 2y - y^2 \Big]_{\frac{1-u}{2}}^{1}$$

$$= 1 - \left[2\left(\frac{1-u}{2}\right) - \left(\frac{1-u}{2}\right)^2\right] = 1 - \left[(1-u) - \frac{(1-u)^2}{4}\right]$$

$$F_{U_2}(u) = \begin{cases} 0, & u < -1 \\ 1 - \left[(1-u) - \frac{(1-u)^2}{4}\right], & -1 \le u \le 1 \\ 1, & u > 1 \end{cases}$$

$$f_{U_2}(u) = \begin{cases} 1 - \left(\frac{1-u}{2}\right), & -1 \le u \le 1 \\ 0, & \text{else} \end{cases}$$

$$f_{U_2}(u) = \frac{d}{du} F_{U_2}(u)$$

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Q1(c) $U_3 = Y^2$

$$P(U_3 \le u) = P(Y^2 \le u)$$

$$= P(Y^2 \le u \mid Y \ge 0) P(Y \ge 0) + P(Y^2 \le u \mid Y < 0) P(Y < 0)$$

1 0

For $A \subset B$, $P(B) = 1$:

$$P(A \mid B) = \frac{P(A \cap B)}{\cancel{P(B)}} = P(A)$$

1

$$= P(Y^2 \le u \mid Y \ge 0)$$

$$= \begin{cases} 0, & u < 0 \\ P(0 \le Y \le \sqrt{u} \mid Y \ge 0) = P(0 \le Y \le \sqrt{u}), & 0 \le u \le 1 \\ 1, & u > 1 \end{cases}$$

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On $0 \le u \le 1$

$$P(U_3 \le u) = P(0 \le Y \le \sqrt{u}) = \int_0^{\sqrt{u}} 2(1-y)\,dy = 2y - y^2 \Big]_0^{\sqrt{u}}$$

$$= 2\sqrt{u} - u$$

$$P(U_3 \le u) = \begin{cases} 0 & , u < 0 \\ 2\sqrt{u} - u & , 0 \le u \le 1 \\ 1 & , u > 1 \end{cases}$$

$$f_{U_3}(u) = \begin{cases} \frac{1}{\sqrt{u}} - 1 & , 0 \le u \le 1 \\ 0 & , \text{else} \end{cases}$$