STA 237 Tutorial 6 Fall 2021

Q1] $n = 10000$ $p = 0.5$

$X \sim B(10000, \tfrac{1}{2})$

$\mu = np = 10000 \times \frac{1}{2} = 5000$

$\sigma^2 = np(1-p)$

$= 5000 \times \frac{1}{2} = 2500$

So, $\sigma = \sqrt{2500} = 50$

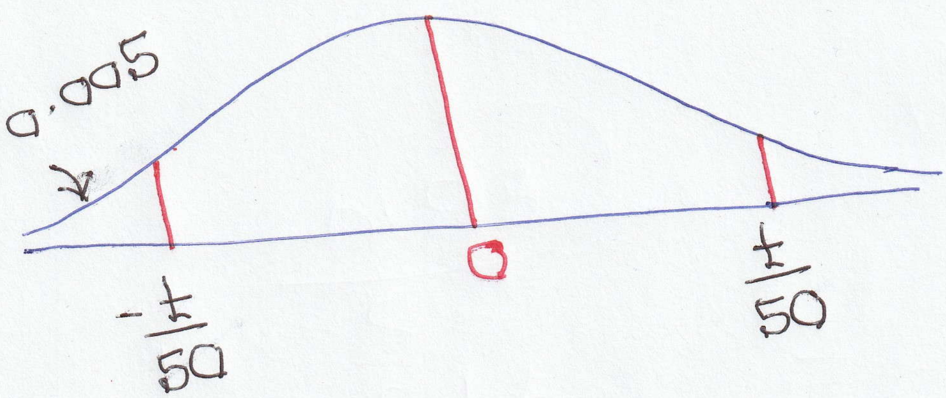
X is approximated by a normal distribution

n is fairly large, so we omit continuity correction

$P(5000 - t \le X \le 5000 + t) = 0.99$

$P\left\{ \frac{-t}{50} \le Z \le \frac{t}{50} \right\} = 0.99$

$1 - 0.99 = 0.01$



$$P\left(Z \le -\frac{t}{50}\right) = 0.005$$

$$-\frac{t}{50} = -2.575$$

$$\Rightarrow \quad t = 128.75 \approx 129$$

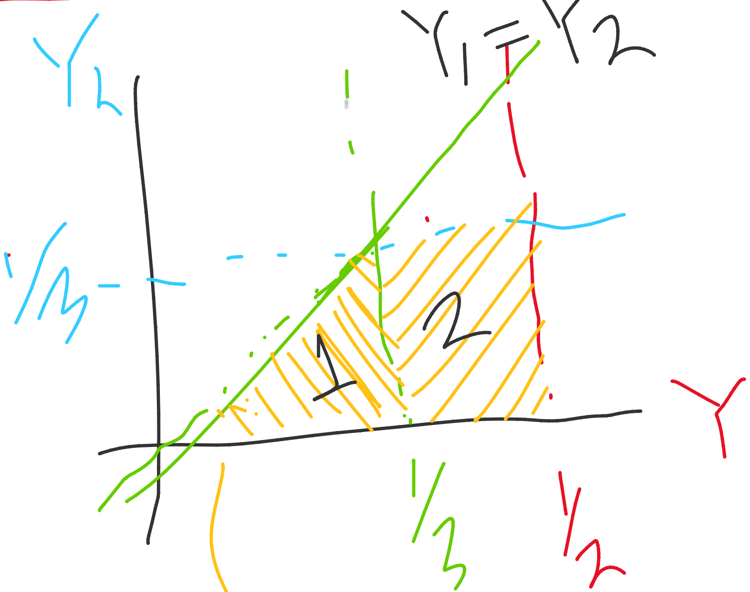
So, $5000 - 129 = 4871$

$5000 + 129 = 5129$

A claim of tossing a fair coin 10000 times and obtaining less than 4871 or more than 5129 heads would be highly suspect.

Q2 (a)

$$P\left(Y_1 < \tfrac{1}{2},\ Y_2 < \tfrac{1}{3}\right) = \text{Region } 1 + \text{Region } 2$$



$0 \le y_2 \le y_1 \le 1$

$$\text{Region } 2 = \int_{1/3}^{1/2}\left[\int_0^{1/3} 3y_1 \, dy_2\right] dy_1$$

$$\int_0^{1/3} 3y_1 \, dy_2 = y_1$$

$$\int_{1/3}^{1/2} y_1 \, dy = \frac{1}{2}\left(\frac{1}{2}^2 - \frac{1}{3}^2\right) = 5/72$$

October 27, 2021 4:41 PM

Region 1

$$\int_0^{1/3}\left[\int_0^{y_1} 3y_1\,dy_2\right]dy_1$$

$$\rightarrow \int_0^{y_1} 3y_1\,dy_2 = 3y_1^2$$

$$\rightarrow \int_0^{1/3} 3y_1^2\,dy_1 = \left(\frac{1}{3}\right)^3$$

OR

$$\int_0^{1/3}\left[\int_{y_2}^{1/3} 3y_1\,dy_1\right]dy_2$$

$$\rightarrow \int_{y_2}^{1/3} 3y_1\,dy_1 = \frac{3}{2}\left(\left(\frac{1}{3}\right)^2 - y_2^2\right)$$

$$\rightarrow \int_0^{1/3} \frac{3}{2}\left(\left(\frac{1}{3}\right)^2 - y_2^2\right)dy_2$$

$$= \frac{3}{2}\left(\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^3\Big/3\right) = \left(\frac{1}{3}\right)^3$$

October 27, 2021 4:49 PM

Region 1 + Region 2

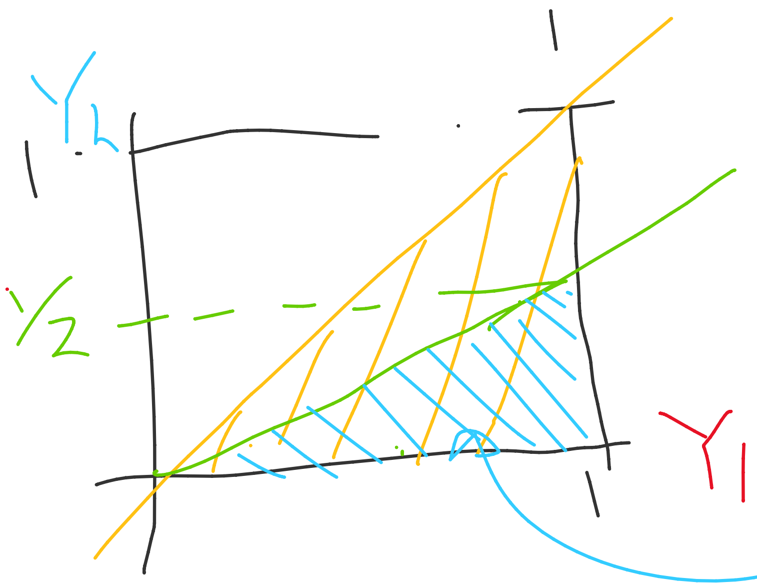
$$= \left(\frac{1}{3}\right)^3 + \frac{5}{72}$$

$$\approx 10.64815\%$$

October 27, 2021 5:02 PM

Q2 (b)

$Y_2 = Y_1$



$Y_2 = \frac{Y_1}{2}$

$\boxed{Y_2 \leq Y_1/2}$

$0 \leq y_2 \leq y_1 \leq 1$

$$\text{Ans} = \int_0^1 \left[\int_0^{y_1/2} 3y_1 \, dy_2 \right] dy_1$$

$$\to \int_0^{y_1/2} 3y_1 \, dy_2 = \frac{3}{2} y_1^2$$

$$\to \int_0^1 \frac{3}{2} y_1^2 \, dy_1 = 1/2$$

October 27, 2021 5:18 PM

Q3

(a) $P(Y_1 = y_1 \mid Y_2 = y_2) = \dfrac{f(y_1, y_2)}{f(y_2)}$

$$f(y_2) = \int_0^{y_2} \left(\frac{1}{2}\right) dy_1 = \frac{y_2}{2} \quad (0 \le y_2 \le 2)$$

$$\Rightarrow P(Y_1 = y_1 \mid Y_2 = K) = \begin{cases} 1/K, & 0 \le y_1 \le K \le 2 \\ 0, & \text{else} \end{cases}$$

* Treat K as constant

October 27, 2021 5:38 PM

(b) Y_1 = dispense (sold)

Y_2 = supply

$$P(Y_1 = y_1 \mid Y_2 = 1.5) = \begin{cases} \frac{1}{1.5}, & 0 \le y_1 \le 1.5 \le 2 \\ 0, & \text{else} \end{cases}$$

$$\rightarrow P(Y_1 \le \tfrac{1}{2} \mid Y_2 = 1.5) = \int_0^{1/2} \frac{1}{1.5}\, dy_1$$

$$= \frac{1}{1.5}\left(\frac{1}{2}\right) = 1/3$$