

Q1

(i) $3X$

Mean 10

$$E(3X) = 3E(X) = 30$$

(ii) $Y+6$

$$E(Y+6) = E(Y) + 6 = 20 + 6 = 26$$

SD 2

$$V(3X) = 3^2 V(X) = (6)^2$$
$$\rightarrow sd(3X) = 6$$
$$V(Y+6) = V(Y) = 5^2$$
$$\rightarrow sd(Y+6) = 5$$

Q1

(iii) $X+Y$

Mean

$$\begin{aligned} E(X+Y) &= E_X + E_Y \\ &= 10 + 20 \\ &= 30 \end{aligned}$$

(iv) $X-Y$

$$\begin{aligned} E(X-Y) &= E_X - E_Y \\ &= 10 - 20 \\ &= -10 \end{aligned}$$

SD ($X \perp Y$)

$$\begin{aligned} V(X+Y) &= V(X) + V(Y) \\ &= 2^2 + 5^2 \\ &= 29 \quad (\text{sd} = \sqrt{29}) \end{aligned}$$

$$\begin{aligned} V(X-Y) &= V(X + (-1)Y) \\ &= V(X) + (-1)^2 V(Y) \\ &= V(X) + V(Y) \\ &= 29 \quad (\text{sd} = \sqrt{29}) \end{aligned}$$

Q1 (v)

* $X_1 \stackrel{D}{=} X_2 \stackrel{D}{=} X$, ** $X_1 \perp X_2$

$X_1 + X_2$

$$\begin{aligned}
 \text{Mean} = E(X_1 + X_2) &= E(X_1) + E(X_2) \\
 &\stackrel{(*)}{=} E(X) + E(X) \\
 &= 10 + 10 = 20
 \end{aligned}$$

$$\begin{aligned}
 \text{Var} = V(X_1 + X_2) &\stackrel{(**)}{=} V(X_1) + V(X_2) \\
 &\stackrel{(*)}{=} V(X) + V(X) \\
 &= 2^2 + 2^2 = 8 \quad (\text{sd} = \sqrt{8})
 \end{aligned}$$

Q4

$$E(\text{profit/policy}) = E \left(\begin{array}{l} \text{Premium/policy} \\ - \text{Admin Fees/policy} \\ - \text{payout/policy} \end{array} \right)$$

(Constant)
(Constant)
(Random)

$$50 = \text{Premium} - 15 - E(\text{payout/policy})$$

$$\Rightarrow \text{Premium} = 50 + 15 + E(\text{payout/policy})$$

$$\text{Payout/policy} = \begin{cases} 1000 & \text{w.p. } \frac{2}{100} \\ 0 & \text{w.p. } 1 - \frac{2}{100} \end{cases}$$

$$\Rightarrow E(\text{payout/policy}) = 1000 \left(\frac{2}{100} \right) = 20$$

$$\therefore \text{Premium} = 50 + 15 + 20 \\ = 85$$